

- [28] S. Yang and J. C. Belfiore, "Optimal space-time codes for the MIMO amplify-and-forward cooperative channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 647–663, Feb. 2007.
- [29] S. Gündüz, M. A. Khojastepour, A. Goldsmith, and H. V. Poor, "Multi-hop MIMO relay networks: Diversity-multiplexing tradeoff analysis," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1738–1747, May 2010.
- [30] M. K. Simon and M. S. Alouini, *Digital Communication Over Fading Channels*. New York, NY, USA: Wiley, 2000.
- [31] A. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Stat.*, vol. 35, no. 2, pp. 475–501, 1964.
- [32] P. Li, D. Paul, R. Narasimhan, and J. Cioffi, "On the distribution of SINR for the MMSE MIMO receiver and performance analysis," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 271–286, Jan. 2006.

## Impact of Primary Network on Secondary Network With Generalized Selection Combining

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**Abstract**—We propose cognitive spectrum sharing with generalized selection combining (GSC) at the secondary user (SU) in the presence of multiple primary transceivers with outdated channel information. Our main motivation is to determine the impact of GSC and outdated channel information on the outage probability of cognitive spectrum sharing subject to two practical power constraints: 1) maximum transmit power at the SU transmitter and 2) peak interference temperature at the primary user (PU) receiver. We derive new closed-form expressions for the exact and asymptotic outage probability in Rayleigh fading. Our expressions provide concise representations of the diversity order and the array gain. We confirm that the diversity order of GSC is entirely dependent on the secondary network and is equal to the available number of receiver antennas at the SU. This result is consistent with those of maximal-ratio combining (MRC) and selection combining (SC) in cognitive spectrum sharing. More importantly, our results show that the outage probability decreases with the increasing correlation coefficient of the outdated channel.

**Index Terms**—Diversity combining, generalized selection combining (GSC), outdated channel information, spectrum sharing.

### I. INTRODUCTION

Cognitive radio has been proposed as a promising means to cope with the rapid growth of volume-intensive bandwidth-hungry mobile applications such as online gaming, social networking, and high-definition video streaming. The so-called underlay cognitive radio allows the secondary user (SU) and the primary user (PU) to transmit

concurrently in the same spectrum provided that the interference from the SU to the PU does not exceed a predetermined interference threshold [1], [2]. The challenge is to effectively coordinate the transmit power at the SU transmitter (SU-Tx), the interference power at the PU receiver (PU-Rx) from the SU-Tx, and the interference power at the SU receiver (SU-Rx) from the PU transmitter (PU-Tx). While cognitive radio increases the reliability of the secondary network, interference requirements in the primary network are often contradictory [3]. The goal of this paper is therefore to address this delicate balance by proposing generalized selection combining (GSC) at the SU-Rx.

To maximize the link reliability of the secondary network and minimize the interference at the primary network, receive diversity at the SU has been applied in interference-limited spectrum sharing networks. While the outage probability and the bit error rate of cognitive networks with a single antenna at the SUs and the PUs were derived in [4] and [5], the capacity of the more general case of multiple antennas at the SU was considered in [6] and [7]. In [6] and [8], maximal-ratio combining (MRC) was applied at the SU. It was shown in [8] that the capacity of cognitive radio with average interference power constraint increases with MRC. In [6], the impact of peak interference power constraint on the capacity was characterized. Considering GSC at the SU, in [7], the capacity with a peak interference power constraint is evaluated. Apart from capacity, the outage probability is another important metric to comprehend the fundamental limits of cognitive radio [9], [10]. The practical consideration of interference from PU-Tx to SU-Rx has been recently examined in [11]. However, the studies in [4]–[11] have all assumed that the SU has full channel state information (CSI) between the SU-Tx and PU-Rxs, and the impact of GSC in the presence of multiple primary transceivers is less well understood.

In this paper, we view cognitive radio from the viewpoint of GSC as a low-power design in interference-limited spectrum sharing networks under outdated CSI between the SU-Tx and PU-Rxs. We consider a general scenario where the SU transmits in the presence of  $M$  PU-Txs and  $M$  PU-Rxs. The main objective is to enhance the transmission reliability of the secondary network while strictly meeting the interference constraint at the primary network, which can be achieved by varying the number of combined antennas at the SU-Rx using GSC. Compared with other diversity combining techniques, GSC is also capable to combat channel estimation errors by eliminating weaker signal-to-interference ratios (SIRs) [12]. To answer some of the pressing questions that face spectrum sharing networks, we relate the maximum transmit power at the SU-Tx with the peak interference power at the PU-Rxs and the interference power from the PU-Txs. Different from previous studies, with GSC, the SU-Rx is allowed to select a subset of its receiver antennas to balance the transmit power at SU-Tx and the interference power at PU-Rx in the presence of multiple PU transceivers. By doing so, less transmit power at the SU-Tx is required to achieve the same outage performance, which in turn reduces the interference at the PU-Rxs. Additionally, a flexible back-off power control mechanism is adopted to guarantee the quality of service (QoS) of the primary network. Owing to this fact, our outcomes are presented in the form of new expressions for the exact and the asymptotic outage probability. Our results bridge the gap between MRC and selection combining (SC) by allowing the SU-Rx to combine the  $L_c$  strongest out of its  $L$  available receiver antennas. We show that the full diversity order is achieved. The diversity order is entirely determined by the secondary network and is equal to  $L$ . We also present the performance gap between GSC, MRC, and SC as a simple ratio of their respective array gains. An interesting conclusion is reached that  $M$  imposes a negative impact on the outage probability, whereas  $L_c$  has a positive impact on the outage probability. Furthermore, the outage probability decreases with an increasing correlation coefficient of the outdated CSI channel.

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## II. NETWORK AND CHANNEL DESCRIPTION

We consider a realistic underlay spectrum sharing network with one pair of SU transceiver and  $M$  pairs of PU transceivers. In the secondary network, we assume a receiver diversity where the SU-Rx is equipped with  $L$  receiver antennas. The  $L_c$  receiver antennas are selected based on the channel from the SU-Tx to the SU-Rx. All other terminals are equipped with only one antenna. We assume that only partial channel knowledge of the interference channel from the SU-Tx to the PU-Rxs is available at the SU-Tx. The CSI of  $h_{1m}$  provided to the SU-Tx is outdated due to the time-varying nature of the wireless link [13]. We describe the outdated CSI of the SU-Tx to the  $m$ th PU-Rx channel using the following correlation model [14], [15]:

$$h_{1m} = \rho \hat{h}_{1m} + \sqrt{1 - \rho^2} \varepsilon \quad (1)$$

where  $m \in \{1, \dots, M\}$ ,  $\hat{h}_{1m}$  is the outdated channel information available at the SU-Tx, and  $\varepsilon$  is a complex Gaussian random variable with zero mean and unit variance and uncorrelated with  $h_{1m}$ . The correlation coefficient  $\rho$  is a constant, which is used to evaluate the impact of channel estimation error, mobility, and feedback delay on the CSI [15]. Note that we assume that both the outdated channel information  $\hat{h}_{1m}$  and the correlation coefficient  $\rho$  are available at SU-Tx.

We assume that both the secondary channel and the primary channel are subject to quasi-static fading where the channel coefficients are constant for each transmission block but vary independently between different blocks. The primary and secondary networks are subject to independent and identically distributed (i.i.d.) Rayleigh fading. The estimated channel power gain from the SU-Tx to the  $m$ th PU-Rx channel is an exponentially distributed RV with parameter  $1/\hat{\alpha}_1$ . The instantaneous channel power gains from the SU-Tx to the  $m$ th PU-Rx channel, from the  $m$ th PU-Tx to the  $l$ th transmit antenna at SU-Rx, and from the SU-Tx to the  $l$ th receiver antenna at SU-Rx are exponentially distributed random variables (RVs) with parameters  $1/\alpha_1$ ,  $1/\alpha_2$ , and  $1/\alpha_3$ , respectively, where  $l \in \{1, \dots, L\}$  and  $\alpha_1 = \rho^2 \hat{\alpha}_1 + (1 - \rho^2)$ . Note that, here, we mainly focus on the outdated channel effect and the impact of GSC; thus, the different path loss and shadowing effect between the SU-Tx and PU-Rxs links resulting from the geometry are not taken into consideration in this paper. This assumption is also applicable to the scenario where PU-Rxs are located in a compact and small area.

### A. Interference Outage Probability

According to underlay spectrum sharing, the interference from the secondary network impinged on the primary network should remain below a predefined peak interference temperature  $Q$  [16]. When only the outdated interference channel information is available at SU-Tx, this strict interference requirement cannot be satisfied at all times. Therefore, the PU-Rxs should tolerate outages occurring for a certain percentage of time, which is named as the interference outage. In [15], it was been proved that the interference outage probability is always 0.5, as long as the maximum transmission power is ignored. Therefore, instead of the strict peak interference constraint, a more flexible constraint known as the power margin, which is based on the interference outage probability is considered to prevent the primary transmission from degradation. The transmit power and the interference outage probability are denoted as

$$P(\rho, |\hat{h}_{1m^*}|^2) = \min \left( P_T, k_I \frac{Q}{|\hat{h}_{1m^*}|^2} \right) \quad (2)$$

$$P_o = 1 - \Pr \left\{ \min \left( P_T, k_I \frac{Q}{|\hat{h}_{1m^*}|^2} \right) |h_{1m^*}|^2 \leq Q \right\} \quad (3)$$

respectively, where  $P_T$  is the maximum available transmit power at the SU-Tx, and  $|\hat{h}_{1m^*}|^2$  is the largest estimated channel power gain from the SU-Tx to the  $M$  PU-Rxs available at SU-Tx. We assume the power margin factor  $k_I = 1$  for the perfect CSI between the SU-Tx and PU-Rx link. Since the closed-form expression for the power margin factor  $k_I$  is intractable [14], therein, we numerically evaluate  $k_I$  based on (3).

## III. SIGNAL-TO-INTERFERENCE RATIO

In the secondary network, GSC is applied to combine the  $L_c$  ( $1 \leq L_c \leq L$ ) antennas with the strongest received signals from  $L$  available receiver antennas at the SU-Rx. The channel coefficients from the SU-Tx to the  $l$ th receiver antenna at the SU-Rx are denoted as  $g_l$  ( $1 \leq l \leq L$ ). Let  $|g_1|^2 \geq |g_2|^2 \geq \dots \geq |g_L|^2 \geq 0$  be the order statistics obtained by sorting  $\{g_l\}_{l=1}^L$  in decreasing order of magnitude [7]. After performing GSC [17], the received signal at the SU-Rx is given by

$$y = \frac{\mathbf{G}^H}{\|\mathbf{G}\|} \mathbf{G} x_T + \underbrace{\sum_{m=1}^M \frac{\mathbf{G}^H}{\|\mathbf{G}\|} \mathbf{H}_{2m} x_m}_{\text{Interferences from PU-Txs}} + \frac{\mathbf{G}^H}{\|\mathbf{G}\|} \mathbf{n} \quad (4)$$

where  $x_T$  is the transmit signal at the SU-Tx,  $\mathbf{G} = [g_1, g_2, \dots, g_{L_c}]^T$  is the selected channel vector at the SU-Rx,  $x_m$  is the transmit signal at the  $m$ th PU-Tx,  $\mathbf{H}_{2m}$  is the  $L_c \times 1$  channel vector from the  $m$ th PU-Tx to  $L_c$  antennas with strongest receive signals at the SU-Rx, and  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{L_c})$  is the  $L_c \times 1$  additive white Gaussian noise vector. Similar to some practical applications such as cellular code-division multiple-access networks and dense ad hoc networks [18], we consider the interference-limited scenario of cognitive networks with multiple primary transceivers where the noise is typically negligible compared with the sum of interference from the multiple PU-Txs. As such, the SIR can be used interchangeably with the signal-to-interference-plus-noise ratio (SINR) without much appreciable loss of accuracy [19]–[21]. Based on (4), the instantaneous received SIR is given by

$$\tilde{\gamma} = \min \left( P_T, \frac{k_I Q}{Y} \right) \frac{X}{V} \quad (5)$$

where  $X = \|\mathbf{G}\|^2$ ,  $Y = |\hat{h}_{1m^*}|^2$ , and  $V = \sum_{m=1}^M P_I |\mathbf{G}^H \mathbf{H}_{2m}| / \|\mathbf{G}\|^2$  is the interference power from the PU-Txs to the SU-Rx.

## IV. OUTAGE PROBABILITY

### A. Exact Analysis

Outage probability is an important metric to characterize the performance of cognitive networks in practice. In cognitive networks, an outage occurs if the instantaneous received SIR at the SU drops below a given threshold  $\gamma_{th}$ . Based on this, the outage probability is formulated as

$$P_{out} = \Pr\{\tilde{\gamma} \leq \gamma_{th}\} = F_{\tilde{\gamma}}(\gamma_{th}) \quad (6)$$

where  $\Pr\{\cdot\}$  denotes the probability, and  $F_{\tilde{\gamma}}(\gamma_{th})$  denotes the cumulative distribution function (cdf) of  $\tilde{\gamma}$ . To derive  $F_{\tilde{\gamma}}(\gamma_{th})$ , we first present the statistics of the channel power gain  $\|\mathbf{G}\|^2$  as follows.

The cdf of  $\|\mathbf{G}\|^2$  is obtained using [17, Eq. (4)] and the multinomial expansion [22] that results in

$$F_{\|\mathbf{G}\|^2}(x) = \sum_{S_k \in S_K} \alpha_k x^{\beta_k} e^{-\delta_k \frac{x}{\alpha_3}} \quad (7)$$

where  $S_K = \{S_k | \sum_{l=0}^L m_{k,l} = 1\}$  with  $\{m_{k,l}\} \in Z^+$ . We note that  $S_K$  refers to all possible combinations of  $\{m_{k,l}, l = 0, 1, \dots, L\}$  that satisfy  $\sum_{l=0}^L m_{k,l} = 1$ . The parameters in (7) are defined as

$$\alpha_k = (\varepsilon_0)^{m_{h,0}} \prod_{l=1}^{L_c} \left( \frac{\varepsilon_l}{\Gamma(l)} \right)^{m_{h,l}} \prod_{l=L_c+1}^L \varepsilon_l^{m_{h,l}} \quad (8)$$

$$\beta_k = \sum_{l=1}^{L_c} (l-1)m_{k,l} \quad (9)$$

$$\delta_k = \sum_{l=1}^{L_c} m_{k,l} + \sum_{l=L_c+1}^L \frac{l}{L_c} m_{k,l} \quad (10)$$

where  $\varepsilon_l$  is given by

$$\varepsilon_l = \begin{cases} 1, & l=0 \\ \alpha_3^{1-l} \left[ -1 + \sum_{k=L_c+1}^L (-1)^{k-l} \frac{\binom{L-k}{L_c-l+1} \binom{k-1}{L_c-l+1}}{\binom{k-1}{L_c-l+1}} \right], & 1 \leq l < L_c \\ -\alpha_3^{1-L_c} \frac{\binom{L}{L-L_c}}{\binom{L-L_c}{L-L_c}}, & l=L_c \\ \frac{(-1)^l \binom{L}{L-l} \binom{l-1}{L-L_c-1}}{\binom{l-1}{L_c-1}}, & L_c < l \leq L. \end{cases} \quad (11)$$

The cdf of  $|\hat{h}_{1m^*}|^2$  is given by

$$F_{|\hat{h}_{1m^*}|^2}(x) = \left( 1 - e^{-\frac{x}{\alpha_1}} \right)^M. \quad (12)$$

Recall that we define  $V = \sum_{m=1}^M P_I |\mathbf{G}^H \mathbf{H}_{2m} / \|\mathbf{G}\||^2$ . According to [23],  $\mathbf{G}^H \mathbf{H}_{2m} / \|\mathbf{G}\|$  is a zero-mean complex Gaussian vector, which is independent of  $\mathbf{G}^H$  and  $\|\mathbf{G}\|$ , and  $|\mathbf{G}^H \mathbf{H}_{2m} / \|\mathbf{G}\||^2$  follows the chi-square distribution with two degrees of freedom. As such, the probability distribution function (pdf) of  $V$  is given by

$$f_V(x) = \left( \frac{1}{P_I \alpha_2} \right)^M \frac{x^{M-1} e^{-\frac{x}{P_I \alpha_2}}}{\Gamma(M)}. \quad (13)$$

Based on the cdf and pdf of  $X$ ,  $Y$ , and  $V$ , we present a closed-form expression for the outage probability in the following theorem.

*Theorem 1:* The closed-form expression for the outage probability of spectrum sharing networks with GSC and outdated CSI between SU-Tx and PU-Rxs is derived as

$$P_{\text{out}}(\gamma_{\text{th}}) = \frac{\Gamma(\beta_k + M)}{\Gamma(M)} \sum_{S_k \in S_K} \alpha_k \gamma_{\text{th}}^{\beta_k} \Xi(\gamma_{\text{th}}) \quad (14)$$

where

$$\begin{aligned} \Xi(\gamma_{\text{th}}) &= \left( 1 - e^{-\frac{k_I Q}{\alpha_1 P_T}} \right)^M \left( \frac{1}{P_I \alpha_2} \right)^M \left( \frac{1}{P_T} \right)^{\beta_k} \\ &\times \frac{1}{\left( \frac{\delta_k \gamma_{\text{th}}}{P_T \alpha_3} + \frac{1}{P_I \alpha_2} \right)^{\beta_k + M}} + \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \left( \frac{\alpha_3}{\hat{\alpha}_1 \delta_k \gamma_{\text{th}}} \right)^{\beta_k + M} \\ &\times \left( \frac{k_I Q}{P_I \alpha_2} \right)^M \beta_k! e^{-\frac{m k_I Q}{\alpha_1 P_T}} \sum_{i=0}^{\beta_k} \frac{\left( \frac{k_I Q}{P_T} \right)^i}{i!} \left( \frac{m}{\hat{\alpha}_1} \right)^{i+M} \\ &\times \Phi \left( \beta_k + M, M + i; \left( \frac{1}{P_I \alpha_2} + \frac{\delta_k \gamma_{\text{th}}}{\alpha_3 P_T} \right) \frac{\alpha_3 m k_I Q}{\hat{\alpha}_1 \delta_k \gamma_{\text{th}}} \right) \end{aligned} \quad (15)$$

where  $\Gamma(\cdot)$  is the gamma function [24, Eq. (8.310.1)], and  $\Phi(\cdot, \cdot; \cdot)$  is the confluent hypergeometric function [24, Eq. (9.211.4)].

*Proof:* See the Appendix. ■

*Lemma 1:* The outage probability of spectrum sharing networks with perfect CSI and only one pair of primary transceiver and secondary transceiver with single antenna is derived as

$$P_{\text{out}}(\gamma_{\text{th}}) = 1 - \frac{\left( 1 - e^{-\frac{Q}{\hat{\alpha}_1 P_T}} \right)}{\left( \frac{P_I \alpha_2 \gamma_{\text{th}}}{P_T \alpha_3} + 1 \right)} - \frac{\alpha_3 Q}{P_I \alpha_2 \hat{\alpha}_1 \gamma_{\text{th}}} e^{\frac{\alpha_3 Q}{P_I \alpha_2 \hat{\alpha}_1 \gamma_{\text{th}}}} \times \Gamma \left( 0, \left( \frac{1}{P_I \alpha_2} + \frac{\gamma_{\text{th}}}{\alpha_3 P_T} \right) \frac{\alpha_3 Q}{\hat{\alpha}_1 \gamma_{\text{th}}} \right) \quad (16)$$

which is equivalent to [4, Eq. (15)] with  $\sigma = 0$ , and this also demonstrates the generality of our result.

Note that the closed-form expression for the outage probability with perfect CSI can be obtained by substituting  $k_I = 1$  into (14).

## B. Asymptotic Analysis

Here, we characterize the asymptotic behavior of the outage probability in the operating region of high  $P_T$ . Here, we consider that the SU-Tx power is proportional to the PU-Rx interference. This indicates that the diversity order exists when the PU-Rx is able to tolerate a high amount of interference from the SU-Tx. This will benefit the secondary network without violating the transmission at the primary network. With this in mind, we gather a deeper understanding of the effect of power scaling on the outage probability. Similar to [25], we consider  $Q = \mu P_T$ , where  $\mu$  is positive constant.

In the high-SIR regime with  $P_T \rightarrow \infty$ , we apply [24, Eq. (3.354.1)] and [24, Eq. (1.211.1)] to derive the first-order expansion of  $F_{\|\mathbf{G}\|^2}(x)$  as

$$F_{\|\mathbf{G}\|^2}^\infty(x) \approx \frac{1}{L_c^{L-L_c} L_c!} x^L. \quad (17)$$

Substituting (17) into (25), the asymptotic outage probability with proportional interference power constraint is presented in the following theorem.

*Theorem 2:* When  $Q = \mu P_T$ , the asymptotic outage probability of spectrum sharing networks with GSC and multiple PU transceivers as  $P_T \rightarrow \infty$  is derived as

$$P_{\text{out}}^\infty \approx (G_c P_T)^{-G_d} \quad (18)$$

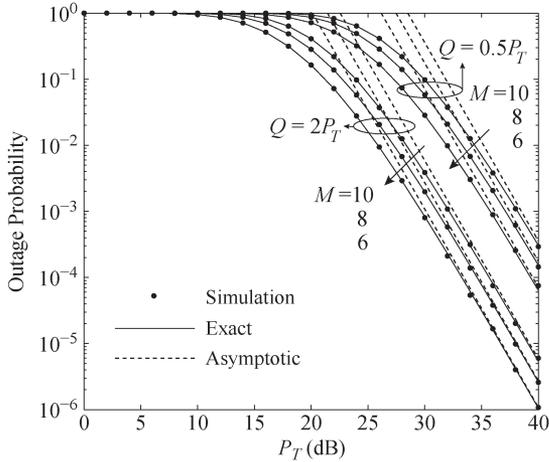
where the diversity order is  $G_d = L$ , and the array gain is

$$G_c = \left[ \frac{\Theta}{L_c^{L-L_c} L_c! \Gamma M} \right]^{-\frac{1}{L}} \frac{1}{\alpha_2 P_I \gamma_{\text{th}}} \quad (19)$$

with  $\Theta = (1 - e^{-(k_I \mu / \hat{\alpha}_1)})^M + \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} (\Gamma(L+1, k_I \mu m / \hat{\alpha}_1)) / (\alpha_3 k_I \mu m / \hat{\alpha}_1)^L$ .

Similarly, the asymptotic outage probability with perfect CSI can be obtained by substituting  $k_I = 1$  into (18). Based on (18), we confirm that the diversity order is entirely determined by the available number of receiver antennas at the SU-Rx  $L$  but independent of the number of combined antennas at the SU-Rx  $L_c$  and the correlation coefficient  $\rho$ .

We now proceed to examine the outage tradeoff between GSC, MRC, and SC. Given that GSC, MRC, and SC maintain the same diversity order, the tradeoff between them is entirely determined by their respective array gains. Based on our results in (19), we present the following remarks.


 Fig. 1. Outage probability with  $L_c = 2$ ,  $L = 3$ ,  $P_0 = 10\%$ , and  $\rho = 0.9$ .

*Remark 1:* We set  $L_c = L$  in (19) to obtain the array gain of MRC as

$$G_c^{\text{MRC}} = \left[ \frac{\Theta \Gamma(M+L)}{L! \Gamma(M)} \right]^{-\frac{1}{L}} \frac{1}{\alpha_2 P_I \gamma_{\text{th}}}. \quad (20)$$

Using (19) and (20), the SNR gap between GSC and MRC is expressed as

$$\Delta_1 = 10 \log \left( \frac{G_c}{G_c^{\text{MRC}}} \right) = \left( \frac{10}{L} \right) \log \left( \frac{L_c^{L-L_c} L_c!}{L!} \right) \text{ dB}. \quad (21)$$

*Remark 2:* We set  $L_c = 1$  in (19) to obtain the array gain of SC as

$$G_c^{\text{SC}} = \left[ \frac{\Theta \Gamma(M+L)}{\Gamma(M)} \right]^{-\frac{1}{L}} \frac{1}{\alpha_2 P_I \gamma_{\text{th}}}. \quad (22)$$

Using (19) and (22), the SIR gap between GSC and SC is expressed as

$$\Delta_2 = 10 \log \left( \frac{G_c}{G_c^{\text{SC}}} \right) = \left( \frac{10}{L} \right) \log (L_c^{L-L_c} L_c!) \text{ dB}. \quad (23)$$

*Remark 3:* Using (20) and (22), the SIR gap between MRC and SC is expressed as follows:

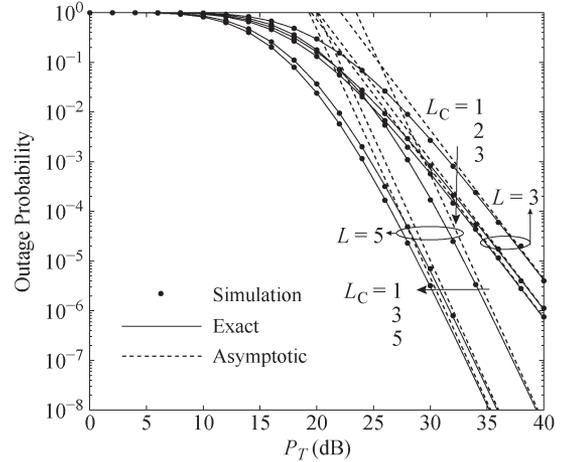
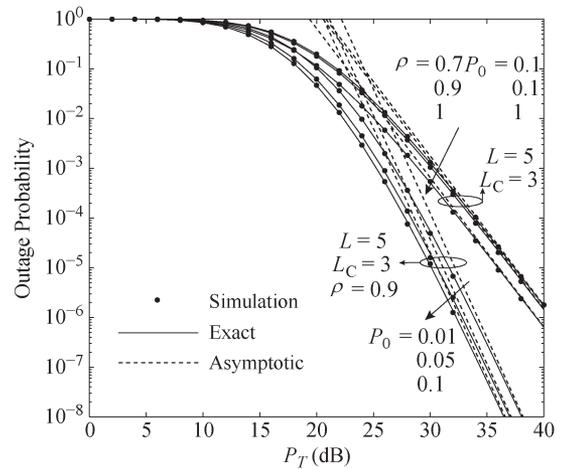
$$\Delta_3 = 10 \log \left( \frac{G_c^{\text{MRC}}}{G_c^{\text{SC}}} \right) = \left( \frac{10}{L} \right) \log(L!) \text{ dB}. \quad (24)$$

Based on *Remarks 1–3*, we confirm that the SIR gaps  $\Delta_1$ ,  $\Delta_2$ , and  $\Delta_3$  are fully described by two parameters: the number of available receiver antennas at the SU-Rx  $L$  and the number of combined antennas at the SU-Rx  $L_c$ .

## V. NUMERICAL RESULTS

Numerical examples are provided to highlight the impact of GSC on the performance of underlay spectrum sharing networks with imperfect CSI. In the examples, we assume that the threshold SIR is  $\gamma_{\text{th}} = 1$  dB, and the interference power from PU-Tx is  $P_I = 3$  dB. We also set  $\hat{\alpha}_1 = 2$ ,  $\alpha_2 = 3$ , and  $\alpha_3 = 1$ . We see a perfect match between the simulations and the exact analytical curves [plotted from (14)]. We also see that the asymptotic analytical lines [plotted from (18)] are in precise agreement with the exact analytical curves in the medium-to-high regime of  $P_T$ .

Fig. 1 plots the outage probability versus  $P_T$  as we vary  $M$ . Here, we set  $L = 3$ ,  $L_c = 2$ ,  $P_0 = 10\%$ , and  $\rho = 0.9$ . We observe that


 Fig. 2. Outage probability with  $M = 6$ ,  $Q = 2P_T$ ,  $P_0 = 10\%$ , and  $\rho = 0.9$ .

 Fig. 3. Outage probability with  $L_c = 3$ ,  $L = 5$ , and  $M = 5$ .

the diversity order is independent of  $M$ , as indicated by the parallel slopes of the asymptotes. The diversity order is entirely determined by the number of antennas at the SU-Rx as  $G_d = L$ . As expected, the outage probability decreases with decreasing  $M$ . This is due to the fact that the array gain in (19) increases with decreasing  $M$ . This result is not surprising since the interference from the PU-Tx to the SU-Rx increases with increasing  $M$ , as shown in (5), which negatively impacts the secondary network. We also observe that setting  $Q = 2P_T$  achieves a lower outage probability compared with  $Q = 0.5P_T$ . This is due to the fact that the array gain in (19) increases with increasing  $\mu$ . In other words, the higher peak interference power constraint at PUs enables more reliable transmission of the secondary network.

Fig. 2 plots the outage probability versus  $P_T$  as we vary  $L_c$ . Here, we set  $M = 6$ ,  $Q = 2P_T$ ,  $P_0 = 10\%$ , and  $\rho = 0.9$ . We observe that the diversity order is  $G_d = L$ , regardless of  $L_c$ . Setting  $L_c = L$  and  $L_c = 1$ , we obtain the special cases of MRC and SC, respectively. We also observe that the SIR gap between MRC and SC are accurately predicted according to (24). As expected, we observe that the outage probability decreases with increasing  $L_c$ . This is due to the fact that the array gain increases with increasing  $L_c$ , as shown in (19). Moreover, we find that the lower outage probability can be achieved by employing more antennas  $L$ , which can be explained by the fact that the array gain increases with increasing  $L$ .

Fig. 3 plots the outage probability versus  $P_T$  as we vary  $\rho$  and  $P_0$ . Here, we set  $M = 5$ ,  $L = 5$ ,  $L_c = 3$ , and  $Q = 1.5P_T$ . We can easily

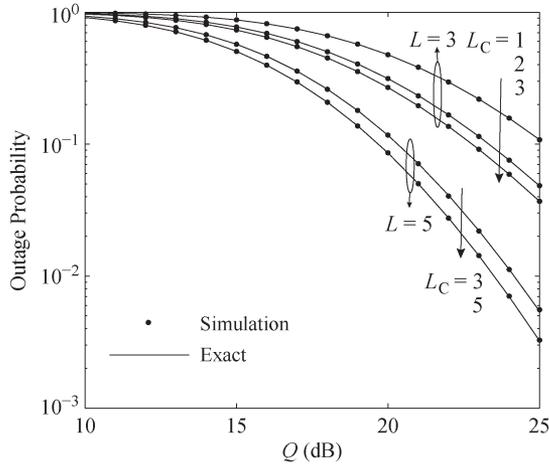


Fig. 4. Outage probability with  $M = 5$ ,  $P_T = 25$  dB,  $P_0 = 10\%$ , and  $\rho = 0.9$ .

see the same parallel slopes for different coefficients  $\rho$ , which indicates that the diversity order is independent of  $\rho$  and  $P_0$ . Interestingly, for the case of  $P_0 = 0.1$ , the outage probability decreases as  $\rho$  increases; for the case of  $\rho = 0.9$ , the outage probability decreases with increasing  $P_0$ . This can be explained by the fact that increasing  $\rho$  and  $P_0$  ensure the higher transmit power of the SU-Tx and thus brings about better outage performance. As expected, for  $\rho = 1$ , which refers to perfect CSI, the lowest outage probability is achieved.

Fig. 4 plots the outage probability versus  $Q$  as we vary  $L_c$ . Here, we set  $M = 5$ ,  $P_T = 25$  dB,  $P_0 = 10\%$ , and  $\rho = 0.9$ . As expected, the outage probability decreases with increasing  $Q$  when  $P_T$  is not a linear function of  $Q$ . Furthermore, the outage probability decreases with increasing  $L_c$ .

## VI. CONCLUSION

We have proposed GSC in interference-limited spectrum sharing networks with outdated CSI and multiple PU transceivers. Our aim is to examine the impact of GSC and outdated channel on the outage probability. To facilitate this, we derived new closed-form expressions for the exact and the asymptotic outage probability. The SU-Rx is allowed to combine the  $L_c$  strongest antennas out of  $L$  antennas. With this in mind, we examined the fundamental question of how  $L_c$ ,  $M$ , and  $\rho$  affect the transmit power at SU-Tx, the interference from SU-Tx to PU-Rx, and the interference from PU-Tx to SU-Rx. The valuable insights are reached that the interference power at the SU-Rx increases as  $M$  increases, and the outage probability is improved with more accurate CSI.

### APPENDIX PROOF OF THEOREM 1

According to (5), the cdf of  $\tilde{\gamma}$  conditioned on  $V$  is written as

$$F_{\tilde{\gamma}|V}(\gamma) = \underbrace{\Pr \left\{ \|\mathbf{G}\|^2 \leq \frac{\gamma V}{P_T}, \|\hat{h}_{1m^*}\|^2 < k_I \frac{Q}{P_T} \right\}}_{I_1(V)} + \underbrace{\Pr \left\{ \frac{\|\mathbf{G}\|^2}{|\hat{h}_{1m^*}|^2} \leq \frac{\gamma V}{k_I Q}, |\hat{h}_{1m^*}|^2 \geq k_I \frac{Q}{P_T} \right\}}_{I_2(V)}. \quad (25)$$

By substituting (7) and (12) into (25), the first term  $I_1(Z)$  is obtained as

$$I_1(V) = \sum_{S_k \in S_K} \alpha_k \left( \frac{\gamma V}{P_T} \right)^{\beta_k} e^{-\frac{\delta_k \gamma V}{\alpha_3 P_T}} \left( 1 - e^{-\frac{k_I Q}{\alpha_1 P_T}} \right)^M. \quad (26)$$

Integrating (26) with respect to the pdf of  $Z$  given by (13) results in

$$E_V \{I_1(V)\} = \left( 1 - e^{-\frac{k_I Q}{\alpha_1 P_T}} \right)^M \left( \frac{1}{P_I \alpha_2} \right)^M \frac{1}{\Gamma(M)} \times \sum_{S_k \in S_K} \alpha_k \left( \frac{\gamma}{P_T} \right)^{\beta_k} \frac{\Gamma(\beta_k + M)}{\left( \frac{\delta_k \gamma}{P_T \alpha_3} + \frac{1}{P_I \alpha_2} \right)^{\beta_k + M}}. \quad (27)$$

Applying the same approach,  $E_V \{I_2(V)\}$  can be derived with the help of [24, Eq. (8.352.2)], [24, Eq. (9.211.4.8)], and the polynomial expansion, which yields

$$E_V \{I_2(V)\} = \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \sum_{S_k \in S_K} \alpha_k \left( \frac{\gamma}{k_I Q} \right)^{\beta_k} \beta_k! \times \left( \frac{1}{P_I \alpha_2} \right)^M \frac{\Gamma(\beta_k + M)}{\Gamma(M)} e^{-\frac{m k_I Q}{\alpha_1 P_T}} \sum_{i=0}^{\beta_k} \frac{\left( \frac{k_I Q}{P_T} \right)^i}{i!} \left( \frac{\alpha_3 k_I Q}{\alpha_1 \delta_k \gamma} \right)^{\beta_k + M} \times \left( \frac{m}{\alpha_1} \right)^{i+M} \Phi \left( \beta_k + M, M + i; \left( \frac{1}{P_I \alpha_2} + \frac{\delta_k \gamma}{\alpha_3 P_T} \right) \frac{\alpha_3 m k_I Q}{\alpha_1 \delta_k \gamma} \right). \quad (28)$$

Substituting (27) and (28) into (25), the cdf of the SIR is finally derived as (14).

## REFERENCES

- [1] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [2] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 471–487, Feb. 2007.
- [3] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.
- [4] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, May 2010.
- [5] Y. Zou, Y.-D. Yao, and B. Zheng, "Outage probability analysis of cognitive transmissions: Impact of spectrum sensing overhead," *IEEE Trans. Wireless Commun.*, vol. 9, no. 8, pp. 2676–2688, Aug. 2010.
- [6] D. Li, "Performance analysis of MRC diversity for cognitive radio systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 849–853, Feb. 2012.
- [7] Q. Wu, Y. Huang, J. Wang, and Y. Cheng, "Effective capacity of cognitive radio systems with GSC diversity under imperfect channel knowledge," *IEEE Commun. Lett.*, vol. 16, no. 11, pp. 1792–1795, Nov. 2012.
- [8] R. Duan, R. Jäntti, M. Elmusrati, and R. Virrankoski, "Capacity for spectrum sharing cognitive radios with MRC diversity and imperfect channel information from primary user," in *Proc. IEEE Global Commun. Conf.*, Miami, FL, USA, Dec. 2010, pp. 1–5.
- [9] T. Q. Duong, D. B. da Costa, T. Tsiftsis, C. Zhong, and A. Nallanathan, "Outage and diversity of cognitive relaying systems under spectrum sharing environments in Nakagami-m fading," *IEEE Commun. Lett.*, vol. 16, no. 12, pp. 2075–2078, Dec. 2012.
- [10] J.-B. Kim and D. Kim, "Outage probability and achievable diversity order of opportunistic relaying in cognitive secondary radio networks," *IEEE Trans. Commun.*, vol. 60, no. 9, pp. 2456–2466, Sep. 2012.

- [11] T. Q. Duong, P. L. Yeoh, V. N. Q. Bao, M. ElKashlan, and N. Yang, "Cognitive relay networks with multiple primary transceivers under spectrum-sharing," *IEEE Signal Process. Lett.*, vol. 19, no. 11, pp. 741–744, Nov. 2012.
- [12] M. Win and J. Winters, "Virtual branch analysis of symbol error probability for hybrid selection/maximal-ratio combining in Rayleigh fading," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1926–1934, Nov. 2001.
- [13] D. S. Michalopoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Amplify-and-forward relay selection with outdated channel estimates," *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1278–1290, May 2012.
- [14] Q. Wu, Z. Zhang, and J. Wang, "Outage analysis of cognitive relay networks with relay selection under imperfect CSI environment," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1297–1300, Jul. 2013.
- [15] H. Kim, H. Wang, S. Lim, and D. Hong, "On the impact of outdated channel information on the capacity of secondary user in spectrum sharing environments," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 284–295, Dec. 2012.
- [16] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.
- [17] X. Cai and G. B. Giannakis, "Performance analysis of combined transmit selection diversity and receive generalized selection combining in Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1980–1983, Nov. 2004.
- [18] T. L. Singal, *Wireless Communications*. New Delhi, India: Tata McGraw-Hill, 2010.
- [19] P. L. Yeoh, M. ElKashlan, K. J. Kim, T. Q. Duong, and G. K. Karagiannidis, "Cognitive MIMO relaying with multiple primary transceivers," in *Proc. IEEE Global Commun. Conf.*, Atlanta, GA, USA, Dec. 2013, pp. 1956–1961.
- [20] S. A. R. Zaidi, M. Ghogho, D. McLernon, and A. Swami, "Achievable spatial throughput in multi-antenna cognitive underlay networks with multi-hop relaying," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 8, pp. 1278–1290, Aug. 2013.
- [21] K. J. Kim, T. Q. Duong, H. V. Poor, and L. Shu, "Performance analysis of cyclic prefixed single-carrier spectrum sharing relay systems in primary user interference," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6729–6734, Dec. 2012.
- [22] R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics*. Reading, MA, USA: Addison-Wesley, 1989.
- [23] H. Q. Ngo, M. Matthaiou, T. Q. Duong, and E. G. Larsson, "Uplink performance analysis of multicell MU-MIMO systems with ZF receivers," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4471–4482, Nov. 2013.
- [24] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York, NY, USA: Academic, 2000.
- [25] T. Q. Duong, D. B. da Costa, M. ElKashlan, and V. N. Q. Bao, "Cognitive amplify-and-forward relay networks over Nakagami-m fading," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2368–2374, Dec. 2012.

## Performance of General STCs Over Spatially Correlated MIMO Single-Keyhole Channels

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**Abstract**—For multiple-input multiple-output (MIMO) Rayleigh channels, it has been shown that transmitter correlations always degrade the performance of general space–time codes (STCs) in high signal-to-noise ratio (SNR) regimes. In this correspondence, however, we show that when MIMO channels experience single-keyhole conditions, the effect of spatial correlations between transmission antennas is more sophisticated for general STCs: When  $M > N$  (i.e., the number of transmission antennas is greater than the number of receiving antennas), depending on how the correlation matrix  $\mathbf{P}$  beamforms the codeword difference matrix  $\Delta$ , the pairwise error probability performance of general STCs can be either degraded or improved in high-SNR regimes. We provide a new measure, which is based on the eigenvalues of  $\Delta$  and the numbers of transmission and receiving antennas, to examine if there exist certain correlation matrices that can improve the performance of general STCs in high-SNR regimes. Previous studies on the effect of spatial correlations over single-keyhole channels only concentrated on orthogonal STCs, whereas our study here is for general STCs and can also be used to explain previous findings for orthogonal STCs. Simulations also show that the results are applicable with imperfect channel knowledge.

**Index Terms**—Keyhole fading, multiple-input multiple-output (MIMO), space–time codes (STCs), spatial correlations.

### I. INTRODUCTION

It is well known that multiple-input multiple-output (MIMO) systems can achieve better spectral efficiency, without requiring extra power consumption and bandwidth expansion. In reality, however, individual antennas could be spatially correlated due to insufficient antenna spacing [1]. The effect of spatial correlation on general space–time codes (STCs) has been extensively studied for Rayleigh fading channels, and it has been shown that in asymptotically high signal-to-noise ratio (SNR) regimes, the transmission correlations always degrade the pairwise error probability (PEP) performance [2], [3], whereas only in the asymptotically low-SNR regimes, the transmission correlations may either improve or degrade the PEP performance [2].

In MIMO channels, if the scattering environment is not so rich, it is demonstrated in [4] and [5] that the channels can experience keyhole conditions where the system only has a rank-1 MIMO channel matrix [6]–[16]. The interest in such keyhole channels has recently increased since they appear in some practically important propagation scenarios. A keyhole scenario where the separation between the transmitting and the receiving ends is due to the 1-D edge diffraction is given in [4]

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